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Axial Thermal Dispersion Conductivity of Open-Cellular Porous Materials

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Nomenclature

c_{pf} = specific heat of fluid at constant pressure, J/kg K
D_n = nominal cell diameter, 0.0254/pores per inch, m
G = incident radiation, W/m^2
$\tilde{g}^*$ = asymmetry factor of the scattering phase function
k = permeability, m^2
k_{da} = axial thermal dispersion conductivity, W/mK
k_{ec} = effective conductive thermal conductivity, W/mK
k_f = thermal conductivity of gas, W/mK
k_t = total effective thermal conductivity, $k_{ec} + k_{da}$, W/mK
Nl = volume of gas at 273.15 K and 1.0133 bar
Pr = Prandtl number
q_{RS} = irradiation on the upper surface of a porous cylinder, W/m^2
Re = Reynolds number defined by $u_m \rho_f \sqrt{k}/\mu_f$
T = temperature, K
T_R = equivalent blackbody temperature, K
T_s = mean temperature of the upper surface of a porous cylinder, K
T_0 = inlet air temperature, K
u_m = mean gas velocity, m/s
w = dimensionless width of a strut with a square cross section, $\frac{1}{2} + \cos[(\frac{1}{3})\cos^{-1}(2\phi - 1) + 4\pi/3]$
z = axial coordinate, m
z_0 = length of a porous cylinder, m
β^* = scaled extinction coefficient, m^{-1}
γ_a = axial thermal dispersion coefficient for open-cell foams
γ_a^* = axial thermal dispersion coefficient for packed-sphere systems
γ_r = radial thermal dispersion coefficient for open-cell foams
γ_r^* = radial thermal dispersion coefficient for packed-sphere systems
μ_f = viscosity, Pa · s
π = ratio of the circumference to its diameter of a circle
ρ_f = density of fluid, kg/m^3
ρ_H = hemispherical reflectivity of the surface of skeletons consisting of an open-cell foam
σ = Stefan–Boltzmann's constant, W/(m^{2}K^4)
ϕ = porosity
ω^* = scaled albedo

Introduction

OPEN-CELLULAR porous materials consisting of pentagonal dodecahedron cells with open-cell walls have been widely utilized as elements of radiators, catalytic converters, regenerative heat exchangers, combustors, and so forth.[1,2] With regard to thermal design and operation of these facilities, fundamental knowledge of the heat transfer characteristics of this kind of porous medium has been needed.[3] A few sets of heat transfer parameters are required in accord with the heat transfer model adopted: When local thermal equilibrium between fluid and solid phases exists within a medium and, hence, the one-temperature model is acceptable as a heat transfer model, only the effective conductive thermal conductivity k_{ec} and axial thermal dispersion conductivity k_{da} are necessary, whereas if the local thermal equilibrium assumption could not be justified and, thus, the two-temperature model must be utilized, the volumetric heat transfer coefficient h_v is also required in addition to the aforementioned parameters. In any case, both k_{ec} and k_{da} are indispensable to heat transfer models of an open-cellular porous medium.

A number of experimental and theoretical studies of k_{ec} have been made during recent decades, and, at present, there exists an accurate theoretical model to predict k_{ec} such as Schuetz–Glicksman's model,[4] but there has been no effort to determine the effective thermal conductivity in the axial direction.[2]

The purpose of the present Note is to remedy this deficiency. To this end, the axial thermal dispersion conductivities of open-cellular condierite–alumina porous materials are determined by an inverse analysis of steady-state axial temperature profiles of the open-cell foams, where radiant energy from infrared lamps flowed countercurrently to the flow of fluid.

Experimental Apparatus and Procedures

The test section for measuring the axial temperature distributions within a porous medium to estimate the axial total effective thermal conductivity consists of a circular steel pipe 0.11 m in inner diameter and 0.17 m in height, where an open-cellular cordierite–alumina porous cylinder 0.1 m in diameter and 0.15 m in height was concentrically placed. A gap between the porous cylinder and the inner wall of the test pipe was filled with clay. The outer surface of the steel pipe was insulated by 0.05-m-thick fiber insulation.

Nine type-K sheathed thermocouples of 0.0016-m diam were inserted through the pipe wall along the central axis of the cylinder to provide information on the axial temperature distribution. In addition, four type-K sheathed thermocouples were settled along the off-central axis, being 0.025 m apart from the central axis, to check radial temperature uniformity within the porous cylinder. Several type-T thermocouple elements were adhered on the upper and lower surfaces of the cylinder (five for the upper surface and two for the lower surface). Inlet air temperature was also measured by type-K sheathed thermocouples.

Four 250-W infrared lamps were used to heat the upper surface of the porous cylinder radiatively and to form an axial temperature gradient within the porous medium. The amount of radiant energy from the infrared lamps was measured on the upper surface of the porous cylinder using a still-type water calorimeter that consists of a 0.1-m-inner diameter and 0.0015-m-thick Bakelite disk with 0.003-m-high rim and 1.3×10^{-5} m thick polyethylene film and that contains 0.025 kg of water. The relative uncertainity in measuring the irradiation was estimated to be ±4%.

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Table 1 Physical characteristics of examined porous media

Material	Porosity ϕ	Nominal cell size D_n, m	Permeability K, m^2	Forschheimer coefficient F
Cordierite–alumina 20	0.877	1.27×10^{-3}	1.18×10^{-8}	0.287
Cordierite–alumina 13	0.830	1.95×10^{-3}	2.64×10^{-8}	0.216
Cordierite–alumina 06	0.813	4.23×10^{-3}	8.93×10^{-8}	0.252

Air at room temperature flowed upward through the test section: The direction of flow and heat are, therefore, opposite to one another. After the infrared lamps were switched on, it took about 60–90 min to attain the steady state of the temperature inside the test section. The voltage of the infrared lamps was regulated manually to keep the surface temperature of the specimen at about 320 K. The temperature variations within the upper surface were found to be from ± 1.1 to ± 5.7 K. The flow rate was varied from 200 to 800 (Nl/min).

In the experiment, three kinds of the cordierite–alumina porous material, whose physical characteristics are summarized in Table 1, were examined. The chemical compositions of the tested porous materials are Al_2O_3/54 wt%, SiO_2/37 wt%, MgO/6 wt%, and the others/3 wt%.

Data Reduction Procedures

To determine the axial thermal dispersion conductivity, we must perform an inverse analysis of the one-temperature or two-temperature energy equations because k_{da} is involved in these equations as parameter. Under the present experimental conditions, the local thermal equilibrium assumption may be justified because, as shown later, the equivalent blackbody temperature of the infrared lamps used is relatively low, and, simultaneously, the extinction coefficient[5] of the open-cellular porous materials is large: 364.2 m^{-1} for the 20 foam, 295.9 m^{-1} for the 13 foam, and 145.9 m^{-1} for the 06 foam.

For this reason, we utilized the following one-temperature energy equation:

$$\rho_f c_{pf} u_m \left(\frac{dT}{dz}\right) = k_t \left(\frac{d^2T}{dz^2}\right) - \beta^*(1-\omega^*)(4\sigma T^4 - G) \qquad (1)$$

The boundary conditions to Eq. (1) are

$$z = 0 : T = T_0, \qquad z = z_0 : T = T_s \qquad (2)$$

Because Eq. (1) involves the incident radiation G, the intensity of radiation field within a medium must also be specified.

Nevertheless, because only the incident radiation and radiative heat flux were necessary for the present analysis, we solved the P_1 equation[6] with respect to G instead of the exact equation of transfer:

$$\frac{d^2G}{dz^2} - 3\beta^{*2}(1-\omega^*)(1-\omega^*\tilde{g}^*)G = -12\beta^{*2}(1-\omega^*)(1-\omega^*\tilde{g}^*)\sigma T^4 \qquad (3)$$

subject to the following boundary conditions:

$$z = 0{:}G(0) - \left\{\frac{2}{3}\bigg/[\beta^*(1-\omega^*\tilde{g}^*)]\right\}\left(\frac{dG}{dz}\right)_{z=0} = 4\sigma T_0^4$$

$$z = z_0{:}G(z_0) + \left\{\frac{2}{3}\bigg/[\beta^*(1-\omega^*\tilde{g}^*)]\right\}\left(\frac{dG}{dz}\right)_{z=z_0} = 4\sigma T_R^4 \qquad (4)$$

where $\tilde{g}^*$ is the asymmetry factor of the scattering phase function of a diffuse sphere, that is, $-\frac{4}{9}$. T_R is an equivalent blackbody temperature of the infrared lamps and was evaluated from

$$T_R = (q_{RS}/\sigma)^{0.25} \qquad (5)$$

In the present study, T_R was found to vary from 600 to 770 K.

The scaled radiative properties[5] were evaluated using the following expressions:

$$\beta^* = \left(\frac{\pi}{4}\right)\frac{(6/\pi)^{\frac{2}{3}}w^2 + \left(4w\big/\sqrt{\pi}\right)(1-w)}{(1-w)D_n}, \qquad \omega^* = \rho_H \qquad (6)$$

where ρ_H is given by the following expression for cordierite–alumina porous materials[7]:

$$\rho_H = 0.698 + 0.26\,(T/1000) \qquad (7)$$

Note that, in Eqs. (1) and (3), only k_t is an unknown parameter and should be estimated from a measured axial temperature distribution $T(z)$ for a given flow rate and given irradiation. To this end, Marquard's method[8] was used. After Eqs. (1) and (3) together with the boundary conditions (2) and (4) were rewritten in dimensionless forms, the dimensionless governing equations were solved numerically using a finite difference method. An optimum value of k_t was determined to minimize a sum of squares of the residuals between predicted and measured temperatures at prescribed axial locations. In the actual computation, the porous region was divided into 148 equally spaced increments.

To check the accuracy of the numerical analyses, 296 equally spaced increments were also adopted in some cases, and the obtained results were compared with the present ones: The comparison showed that the present results are 0.4% greater than the more exact ones at worst.

On the other hand, the uncertainity in using type-K thermocouples was ± 0.4%, and the uncertainty in wire integrity was estimated to be 10% at worst; thus, the resultant uncertainty in measuring temperatures was ± 10.4%. Consequently, the accuracy of both the experimental and numerical method was within 10.8%.

Once k_t $(= k_{ec} + k_{da})$ is determined as a function of u_m, the axial thermal dispersion coefficient γ_a can be obtained by a regression analysis, because, by analogy with the expression for the lateral thermal dispersion conductivity,[9] the axial thermal dispersion conductivity k_{da} may be written as

$$k_{da} = \rho_f c_{pf} \sqrt{k}\, \gamma_a u_m \qquad (8)$$

Moreover, note that, if the effect of radiation is negligible, then Eq. (1) becomes

$$\rho_f c_{pf} u_m \left(\frac{dT}{dz}\right) = k_t \left(\frac{d^2T}{dz^2}\right) \qquad (9)$$

which yields the following analytical solution:

$$T(z) = T_0 \exp(\rho_f c_{pf} u_m z/k_t) \qquad (10)$$

For z close to z_0, Eq. (10) can be approximated by[10]

$$\ln[T(z) - T_0] \approx -(\rho_f c_{pf} u_m/k_t)(z_0 - z) \qquad (11)$$

This means that, when the effect of radiation can be disregarded, $\ln[T(z) - T_0]$ becomes a linear function of $(z_0 - z)$.

Results and Discussion

Typical examples of the axial temperature profiles within cordierite–alumina porous media are shown in Fig. 1, where the solid lines indicate the temperature profiles determined by the inverse analysis, whereas the broken lines indicate least-squares fit of Eq. (11) to measured data of the temperature at several locations ranging from $z_0 - z = 0.009$ to 0.025 m. The measured axial temperature distributions, $T(z) - T_0$, decrease with an increase in $z_0 - z$ and are asymptotic to zero. Equation (11) well approximates the experimental data, which means that the effect of radiation is comparatively weak. Some discrepancies occur between the exact inverted results and the experimental ones, but note that the exact inversion was performed using measured temperature data over the entire region of a porous medium.

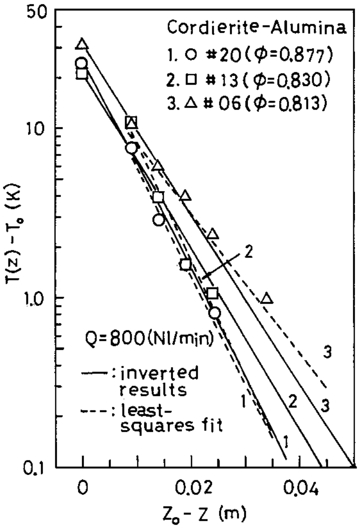

Fig. 1 Examples of experimentally observed axial temperature distributions within cordierite–alumina open-cellular porous materials.

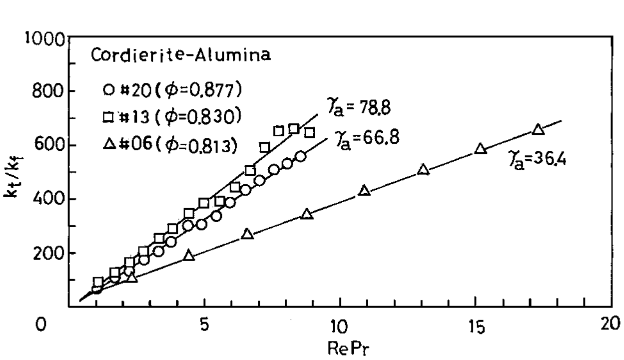

Fig. 2 Variations in the dimensionless axial total effective thermal conductivity against Peclet number.

Results for k_t/k_f are shown in Fig. 2 vs the Peclet number defined by $RePr$. As seen from Fig. 2, k_t/k_f is well approximated by a linear function of $RePr$ and, therefore, use of Eq. (12) is completely justified:

$$k_t/k_f = k_{ec}/k_f + \gamma_a RePr \tag{12}$$

Values of γ_a can be evaluated using a least-squares method, and determined values of γ_a were found to be 36.4 for the 06 foam, 66.8 for the 13 foam, and 78.8 for the 20 foam. The obtained results of γ_a for the 13 and 20 foams are well approximated by the following expression:

$$\gamma_a = 139.91/|\ln(1-\phi)| \tag{13}$$

This formula may be applicable to open-cellular foams with nominal cell diameters smaller than about 0.002 m.

Comparison with the radial thermal dispersion coefficient $\gamma_r (= 0.0877/|\ln(1-\phi)|)$ reveals that the obtained axial thermal dispersion coefficient is about 1600 times greater than the radial one, although the expression for γ_r was obtained using the experimentally determined radial thermal dispersion coefficients of fibrous media with the porosity greater than 0.94 and should be carefully treated in extrapolating it to the porous media with the porosity less than about 0.9 (Ref. 2). In addition, because the axial thermal dispersion coefficient of a packed-sphere system γ_a^* is given by

$$\gamma_a^* = 117.72(1-\phi)^{3.6247}/\phi^{1.5} \tag{14}$$

then γ_a is 181 times greater than γ_a^* at $\phi = 0.8$ and 1857 times greater than γ_a^* at $\phi = 0.9$. This may be attributable to that an open-cellular porous material has a three-dimensional reticulated structure, and, thus, quite strong mixing of fluid occurs within voids, and then the well-stirred fluid is extruded to the neighboring voids through open-cell walls.

Conclusions

The axial thermal dispersion coefficients of open-cellular cordierite-alumina foams, γ_a, were originally determined by the inverse analysis of steady-state axial temperature profiles within the porous media. It is found that γ_a is much greater than the previously reported radial thermal dispersion coefficients of open-cell foams and packed beds and the known axial thermal dispersion coefficients of packed beds.

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